

# Quantum efficiencies in finite disordered networks connected by many-body interactions

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The quantum efficiency in the transfer of an initial excitation in disordered finite networks, modeled by the  $k$ -body embedded Gaussian ensembles of random matrices, is studied for bosons and fermions. The influence of the presence or absence of time-reversal symmetry and centrosymmetry/centrohermiticity are addressed. For bosons and fermions, the best efficiencies of the realizations of the ensemble are dramatically enhanced when centrosymmetry (centrohermiticity) is imposed. For few bosons distributed in two single-particle levels this permits perfect state transfer for almost all realizations when one-particle interactions are considered. For fermionic systems the enhancement is found to be maximal for cases when all but one single particle levels are occupied.

Keywords: Disordered networks, quantum state transfer, many-body interactions, embedded ensembles.

## I. INTRODUCTION

An important question on complex quantum systems, which remains open to a large extent, addresses the conditions to have robust efficient transport of excitations across a disordered finite network [1–3]. Whereas it is well known how to define a Hamiltonian system where perfect transport is obtained [4, 5], the fact that such system requires the precise specification of a large number of parameters makes it difficult to achieve in practice. This is the sense of robustness above: statistical changes in the parameters should lead to small fluctuations that preserve good efficiencies, instead of large transmission fluctuations. Clearly, the number of control parameters should be as small as possible.

Inspired by Ref. [6], we study here the distribution of the transport efficiencies of an initial localized excitation in a disordered network of  $l$  sites, which is modeled by random Hamiltonian that includes many-body interactions, considering both fermions and bosons; to the best of our knowledge, quantum efficiencies of this type of disordered networks have not been considered. In general, this question is of interest in a variety of fields, including understanding photosynthetic light-harvesting complexes [7], such as the Fenna-Matthews-Olson (FMO) complex [8], or in quantum communication protocols across quantum spin chains [9–12]. In either case, a model with few-body interactions seems realistic.

The transport efficiency from an input state  $|\text{in}\rangle$  to an output state  $|\text{out}\rangle$ , is quantified as the maximum transition probability achieved among these states within a time interval  $[0, T]$ . The transport efficiency is defined as [6]

$$\mathcal{P}_{\text{in,out}} = \max_{[0,T]} |\langle \text{out} | U(t) | \text{in} \rangle|^2. \quad (1)$$

Here,  $U(t)$  is the unitary quantum evolution associated with the Hamiltonian of the system and  $T$  is a reasonable time scale ( $\hbar = 1$ ). The system is said to have perfect state transfer (PST) when  $\mathcal{P}_{\text{in,out}} = 1$  [4]. The nodes of the network are the basis states of the Hilbert space where the initial excitation is localized. We define the Hilbert space by distributing  $n$  spinless particles (bosons or fermions) in  $l$  single-particle states. The  $n$ -body Hamiltonian that we consider consists of a random  $k$ -body interaction among the  $n$ -particles states, with  $1 \leq k \leq n$ ; the non-zero Hamiltonian matrix elements are related to the links of the network. This matrix model is known as the (bosonic or fermionic)  $k$ -body embedded Gaussian ensemble of random matrices [13, 14].

The motivation for choosing this matrix model comes from the fact that this ensemble displays correlations among the matrix elements [15, 16], in the sense that the number of independent random variables is usually smaller than the number of independent links of the network. In addition, the bosonic ensemble for the specific case where the bosons are distributed in two single-particle levels displays a systematic appearance of doublets in the spectrum [17]. Thus, one of the design principles required in Ref. [6], the existence of a dominant doublet, is automatic fulfilled. The extension to fermions arises naturally, and is also motivated by the transport processes within the FMO system [7].

As in previous studies [18], centrosymmetry [19] is considered and we find that it increases dramatically the best efficiencies among certain pairs of distinct sites, in comparison to a non-centrosymmetric interaction; we also study in this paper the case where time-reversal symmetry is broken, and the corresponding case including centrohermiticity [20]. Here, as usual, the benchmark for good efficiencies is 95%. In particular, for  $k = 1$  and few bosons distributed in two single-particle states, we find that PST is obtained for a large fraction of the best efficiencies of the ensemble when centrosymmetry or centrohermiticity are included; for larger values of  $k$  the best efficiencies are reduced. In the case of fermions and including centrosymmetry or centrohermiticity, we

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show that below half-filling ( $n/l < 1/2$ , where  $n$  is the number of fermions and  $l$  the number of single-particle states) the best efficiencies are obtained for  $k = 1$ ; above half-filling, our numerics indicate that the best efficiencies correspond to  $k \sim l/2$  for  $n = l - 1$ . In both cases, the benchmark is achieved with non-zero probability.

The paper is organized as follows: in Sect. II we define the matrix model considered and centrosymmetry, which is introduced at the one-particle level and then extended to the  $k$ - and  $n$ -particle spaces. In Sect. III we describe the results for the distribution of the best efficiencies of each realization of the ensemble for bosons occupying two-single particle states, with or without the addition of centrosymmetry or centrohermiticity; Sect. IV analyzes the fermionic case along the same lines. Finally, in Sect. V we present the conclusions of this work.

## II. THE EMBEDDED GAUSSIAN ENSEMBLES AND CENTROSYMMETRY / CENTROHERMITICITY FOR INTERACTING MANY-BODY SYSTEMS

We introduce the  $k$ -body embedded Gaussian ensembles of random matrices for bosons and fermions, for the cases of orthogonal ( $\beta = 1$ ) and unitary ( $\beta = 2$ ) symmetries, following Ref. [13]. We consider a set of  $l$  degenerate single-particle states  $|j\rangle$ , with  $j = 1, 2, \dots, l$ . The associated creation and annihilation operators for fermions are  $a_j^\dagger$  and  $a_j$ , and  $b_j^\dagger$  and  $b_j$  for bosons, with  $j = 1, \dots, l$ . These operators obey the usual (anti)commutation relations that characterize the corresponding particles. We define the operators that create a normalized state with  $k < l$  fermions from the vacuum as  $\psi_{k;\alpha}^\dagger = \psi_{j_1, \dots, j_k}^\dagger = \prod_{s=1}^k a_{j_s}^\dagger$ , with the convention that the indexes are ordered increasingly  $j_1 < j_2 < \dots < j_k$  ( $\alpha$  simplifies the notation for these indexes); the corresponding annihilation operators are  $\psi_{k;\alpha} = (\psi_{k;\alpha}^\dagger)^\dagger$ . Likewise, the  $k$ -boson states are given by  $\chi_{k;\alpha}^\dagger = \chi_{j_1, \dots, j_k}^\dagger = N_\alpha \prod_{s=1}^k b_{j_s}^\dagger$ , where again  $j_1 \leq j_2 \leq \dots \leq j_k$ . Here,  $N_\alpha$  is a factor that guarantees the normalization to unity of  $\chi_{k;\alpha}^\dagger|0\rangle$ : if the index  $j$  repeats  $k_j$  times,  $N_\alpha$  contains a factor  $(k_j!)^{-1/2}$ .

The random  $k$ -body Hamiltonian for fermions reads

$$H_k^{(\beta)} = \sum_{\alpha, \gamma} v_{k;\alpha, \gamma}^{(\beta)} \psi_{k;\alpha}^\dagger \psi_{k;\gamma}; \quad (2)$$

a similar equation holds for bosons replacing  $\psi_{k;\alpha}^\dagger$  by  $\chi_{k;\alpha}^\dagger$ . In Eq. (2) the coefficients  $v_{k;\alpha, \gamma}^{(\beta)}$  are random distributed independent Gaussian variables with zero mean and constant variance

$$\overline{v_{k;\alpha, \gamma}^{(\beta)} v_{k;\alpha', \gamma'}^{(\beta)}} = v_0^2 (\delta_{\alpha, \gamma'} \delta_{\alpha', \gamma} + \delta_{\beta, 1} \delta_{\alpha, \alpha'} \delta_{\gamma, \gamma'}). \quad (3)$$

Here,  $\beta$  is Dyson's parameter that accounts for the presence ( $\beta = 1$ ) or absence ( $\beta = 2$ ) of time-reversal symmetry [21], the bar denotes ensemble average, and we set  $v_0 = 1$  without loss of generality.

The Hamiltonian  $H_k^{(\beta)}$  acts on a Hilbert space spanned by distributing  $n \geq k$  particles on the  $l$  single-particle states. Then, a complete set of basis states is given by the set  $\psi_{n;\alpha}^\dagger|0\rangle$  for fermions (with  $l > n$ ), and  $\chi_{n;\alpha}^\dagger|0\rangle$  for bosons. The dimension of the Hilbert spaces are, respectively,  $N_F = \binom{l}{n}$  and  $N_B = \binom{l+n-1}{n}$ . This defines the  $k$ -body embedded Gaussian ensembles of random matrices [13, 14].

By construction, the case  $k = n$  is identical to the canonical ensembles of Random Matrix Theory [21], i.e., to the Gaussian Orthogonal Ensemble (GOE) for  $\beta = 1$  or the Gaussian Unitary ensemble (GUE) for  $\beta = 2$ . For  $k < n$ , the matrix elements of  $H_k^{(\beta)}$  may be identical to zero and display correlations. The former property appears whenever there are no  $k$ -body operators that link together the  $n$ -body states, e.g., for  $k < n/2$ . Correlations arise because matrix elements of  $H_k^{(\beta)}$  not related by symmetry may be identical. One of the notorious consequences of this is, for the bosonic ensemble in the dense limit ( $n \gg k, l$ , for  $k$  and  $l$  fixed) that the ensemble is non-ergodic [16]. In particular, for  $l = 2$ ,  $k \ll n$  and  $\beta = 1$ , the spectrum displays a significant number of quasi-degenerate states [17].

As mentioned above, centrosymmetry is an important concept for an optimal efficiency [18]. A symmetric  $N \times N$  matrix  $A$  is centrosymmetric if  $[A, J] = 0$ , where  $J$  is the *exchange matrix*  $J_{i,j} = \delta_{i, N-i+1}$  [19]; for complex hermitian matrices, centrohermiticity is defined when  $J A^T J = A$  [20].

Imposing centrosymmetry to the  $k$ -body embedded ensembles is subtle. It can be introduced either at the one-particle level, which is the core for the definition of the  $k$ - and  $n$ -particle Hilbert spaces, or at the  $k$ -body level, where the actual (random) parameters of the embedded ensembles are set, or at the  $n$ -body level, where the system evolves. Considering a more realistic description which includes a one-body (mean-field) term and a two-body (residual) interaction,  $H = H_{k=1} + H_{k=2}$ , it seems unnatural to define a specific transformation for each term separately. Hence, we shall introduce it in the one-particle space, and compute how is it transferred to the  $k$ -body and  $n$ -body space, which depends on  $\beta$  as well. Then, in the one-particle space we define  $J_1|j\rangle = |l-j+1\rangle$  for  $j = 1, \dots, l$ , whose matrix representation in the one-body basis is precisely the exchange matrix. For two fermions, we define  $J_2 \psi_{2;j_1, j_2}^\dagger = J_1 a_{j_1}^\dagger J_1 a_{j_2}^\dagger = -\psi_{2; l-j_2+1, l-j_1+1}^\dagger$ . In the last equality we keep the convention that the indexes are arranged in increasing order; then, the fermionic anticommutation relations impose a global minus sign, which can be safely ignored. This is generalized for  $k$  particles as

$$J_k \psi_{k;j_1, \dots, j_k}^\dagger = \prod_{s=1}^k J_1 a_{j_s}^\dagger = \psi_{k; l-j_k+1, \dots, l-j_1+1}^\dagger, \quad (4)$$

where we have dropped any global minus sign. We note that in general the matrix  $J_k$ , as defined by Eq. (4), is

not an exchange matrix, i.e., the matrix with ones in the counterdiagonal and zeros elsewhere. This follows from the possible existence of more than one state that is mapped by  $J_k$  onto itself; in this case, we shall say that  $J_k$  is a *partial* exchange matrix. As an example, consider the case of fermions distributed in  $l = 4$  single-particle states; for  $k = 2$  the  $k$ -particle space has dimension 6. Then,  $J_2\psi_{2,2,3}^\dagger = \psi_{2,2,3}^\dagger$  and  $J_2\psi_{2,1,4}^\dagger = \psi_{2,1,4}^\dagger$ , ignoring the minus signs mentioned above. Then, the entries in the  $J_2$  matrix elements for these basis states are 1 in the diagonal. In contrast, for the case  $l = 4$  and  $k = 1, 3$  the resulting matrices  $J_1$  and  $J_3$  are exchange matrices.

For bosons  $J_k$  is defined as for fermions using Eq. (4) with the corresponding change of the creation operators. In this case, again,  $J_k$  may not be an exchange matrix. As an example consider  $k = 2$  and  $l = 3$ ; then, we have  $J_2\chi_{2,2,2}^\dagger = N_\alpha(J_1b_2^\dagger)^2 = \chi_{2,2,2}^\dagger$ , and likewise  $J_2\chi_{2,1,3}^\dagger = \chi_{2,1,3}^\dagger$ , which shows that there are more than one basis states mapped onto themselves under  $J_2$ . Yet, for the special case  $l = 2$  that we study below,  $J_k$  is an exchange matrix for all  $k$ , and  $H_k$  (in the  $n$ -particle space) inherits the centrosymmetry (or centrohermiticity) of  $v_k$ , i.e.,  $[J_n, H_k] = 0$ , with  $J_n$  the exchange matrix of appropriate dimensions.

### III. STATISTICS OF THE TRANSPORT EFFICIENCY FOR BOSONS DISTRIBUTED IN $l = 2$ LEVELS

We analyze here the transport efficiency  $\mathcal{P}_{\mu,\nu}$  for bosons distributed in  $l = 2$  single-particle states, considering an arbitrary initial state  $\chi_{n;\mu}^\dagger$  and an arbitrary final one  $\chi_{n;\nu}^\dagger$ . We address separately the cases where the  $k$ -body random interaction is invariant or not with respect to time-reversal ( $\beta = 1$  or  $\beta = 2$ ), and also when centrosymmetry (or centrohermiticity) is or not additionally imposed. In the simulations described below, we considered for concreteness the case with  $n = 9$  bosons, the corresponding Hilbert space dimension is  $N_B = 10$ , the number of realizations of the ensemble is 2000, and  $T = 15$  in Eq. (1).

We discuss first the case where centrosymmetry or centrohermiticity, respectively for  $\beta = 1$  and  $\beta = 2$ , is absent. The distributions of the efficiencies  $\mathcal{P}_{\mu,\nu}$  of the ensemble for any combination of the input and output states is in general rather broad, with negligible contributions to efficiencies above 0.95. More interesting is to focus on the distribution of best efficiencies  $\mathcal{P}$  of each realization of the ensemble. The results are presented in Fig. 1. The empty histograms correspond to  $\beta = 1$  and the shaded ones to  $\beta = 2$ . With respect to time-reversal invariance, the case  $\beta = 2$  seems to yield marginally better efficiencies for a given value of  $k$ , in the sense that the mean value of the distribution attains larger values and the distribution is somewhat narrower. Yet, we notice that the largest values of  $\mathcal{P}$  are slightly dominated by the  $\beta = 1$  case, except for  $k = 1$ . Regarding the depen-

dence on  $k$ , the case  $k = 1$  notably distinguishes itself as a special one. First, the distribution of  $\mathcal{P}$  is dominated by a peak at 0.4, is wider than for other values of  $k$ , and displays the largest number of realizations with efficiencies larger than the benchmark value 0.95 (vertical dashed line). Apart from this case, other values of  $k$  display rather poor efficiencies, with smaller probabilities of reaching the benchmark value if they do it at all. Actually, the distributions for intermediate values of  $k$  ( $4 \leq k \leq 6$ ) seem to be marginally better. In general, the states  $\chi_{n;\mu}^\dagger$  and  $\chi_{n;\nu}^\dagger$  that yield the best efficiencies differ by one boson in the occupation of each single-particle state, except for  $k = 9$  where the best efficiency is uniformly distributed for all pairs of states. This statement is consistent with the fact that  $k = n$  corresponds to a GOE or GUE. Clearly, these unconstrained embedded Gaussian ensembles yield poor transfer efficiencies, except maybe for  $k = 1$ .

We turn now to the best efficiencies of each realization of the ensemble when centrosymmetry or centrohermiticity is imposed. As mentioned above, for the specific bosonic ensemble with  $l = 2$ , imposing centrosymmetry or centrohermiticity at the one-particle level carries over to the  $k$ - and  $n$ -particle spaces; hence,  $[H_k, J_n] = 0$ , with  $J_n$  the  $N_B \times N_B$  exchange matrix. In Fig. 2 we present the best efficiencies  $\mathcal{P}$  of each realization of the ensemble for this case. In comparison to the results presented in Fig. 1, centrosymmetry (centrohermiticity) enhances dramatically the best efficiencies. In particular, the case  $k = 1$  is remarkable since it displays perfect state transfer for *all* realizations of the ensemble for  $\beta = 2$ , while for  $\beta = 1$  there is a small remnant of a peak around 0.4. Quantitatively, for  $\beta = 1$  about 92% of the realizations exhibit efficiencies that are larger than the benchmark value, while for  $\beta = 2$  this number is well above 99%; the mean of the distributions is 0.9556 and 0.9978, respectively. Increasing  $k$  shifts the mean value of the distribution towards smaller values and widens the distributions, being the effect larger for  $\beta = 2$  than for  $\beta = 1$ . Whereas for  $k = 2$  some realizations still display almost PST (last bin of the histograms), this decreases for increasing values of  $k$ . For  $\beta = 1$ , all values of  $k$  exhibit realizations whose best efficiencies are above the benchmark value, with better efficiencies appearing for smaller values of  $k$ .

The states  $\chi_{n;\mu}^\dagger$  and  $\chi_{n;\nu}^\dagger$  that display the best efficiencies are those linked by centrosymmetry, i.e.,  $J_n\chi_{n;\mu}^\dagger = \chi_{n;\nu}^\dagger$ . While for  $k = n$  all centrosymmetry-related pairs participate uniformly, for other values of  $k$  there is a significant dominance of the pair of states where all bosons are initially in either of the single-particle states. From the perspective of the network, these states are located at the edges.

As mentioned above and manifested in Fig. 2(a), the case  $k = 1$  is quite special. Indeed, for  $k = 1$  there are only three ( $\beta = 1$ ) and four ( $\beta = 2$ ) independent random matrix elements; centrosymmetry (centrohermiticity) imposes that they become only two ( $\beta = 1$ ) and three ( $\beta = 2$ ). The consequence of this is that the random

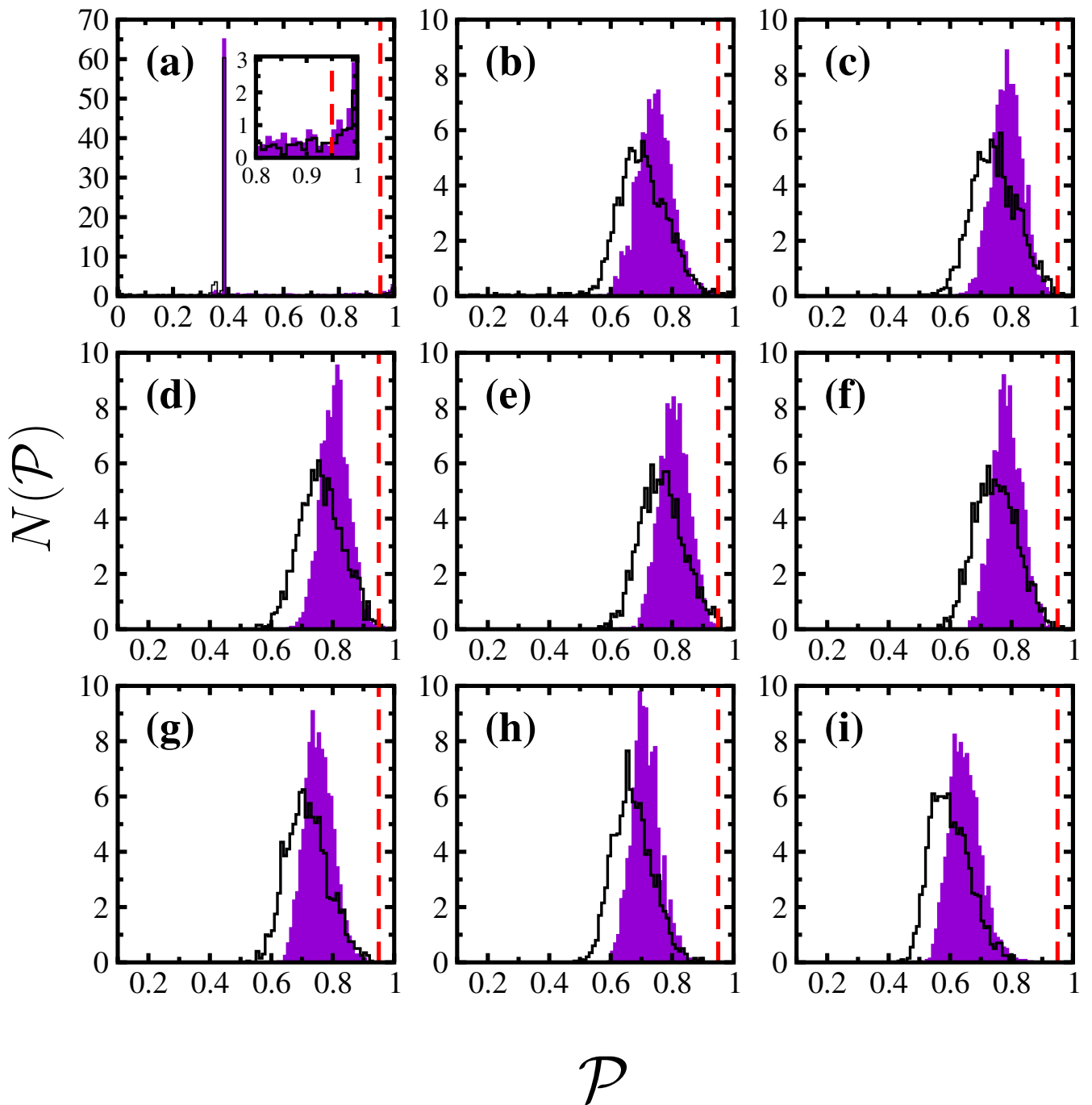


Figure 1. Normalized distributions of the best efficiencies  $\mathcal{P}$  of each realization of the bosonic embedded Gaussian ensemble with no centrosymmetry (or centrohermiticity) for  $l = 2$ ,  $n = 9$ : (a)  $k = 1$ , (b)  $k = 2$ , (c)  $k = 3$ , (d)  $k = 4$ , (e)  $k = 5$ , (f)  $k = 6$ , (g)  $k = 7$ , (h)  $k = 8$ , and (i)  $k = 9$ . The empty histograms correspond to  $\beta = 1$  and the shaded (violet) to  $\beta = 2$ . The red vertical line indicates the 95% benchmark for the efficiency. Note that the scales for the  $k = 1$  case are different from the rest. The inset in (a) is an enlargement of a region close to the benchmark value, showing the non-zero probability of having PST.

many-body centrosymmetric/centrohermitian Hamiltonian has a constant main diagonal, which is proportional to the total number of bosons, and a second-diagonal whose matrix elements are proportional to  $\sqrt{(n_i + 1)(N_B - n_i - 1)}$ , where  $n_i = 0, \dots, n$  is the boson occupation number of one of the single-particle lev-

els for the many-body state in question. Except for the constant diagonal and the random weights involved, for  $\beta = 1$  this matrix model has been shown to exhibit PST [22]. It can be shown that the eigenvectors of both models are identical, and the eigenvalues are related by a linear function. This ensures the occurrence of PST

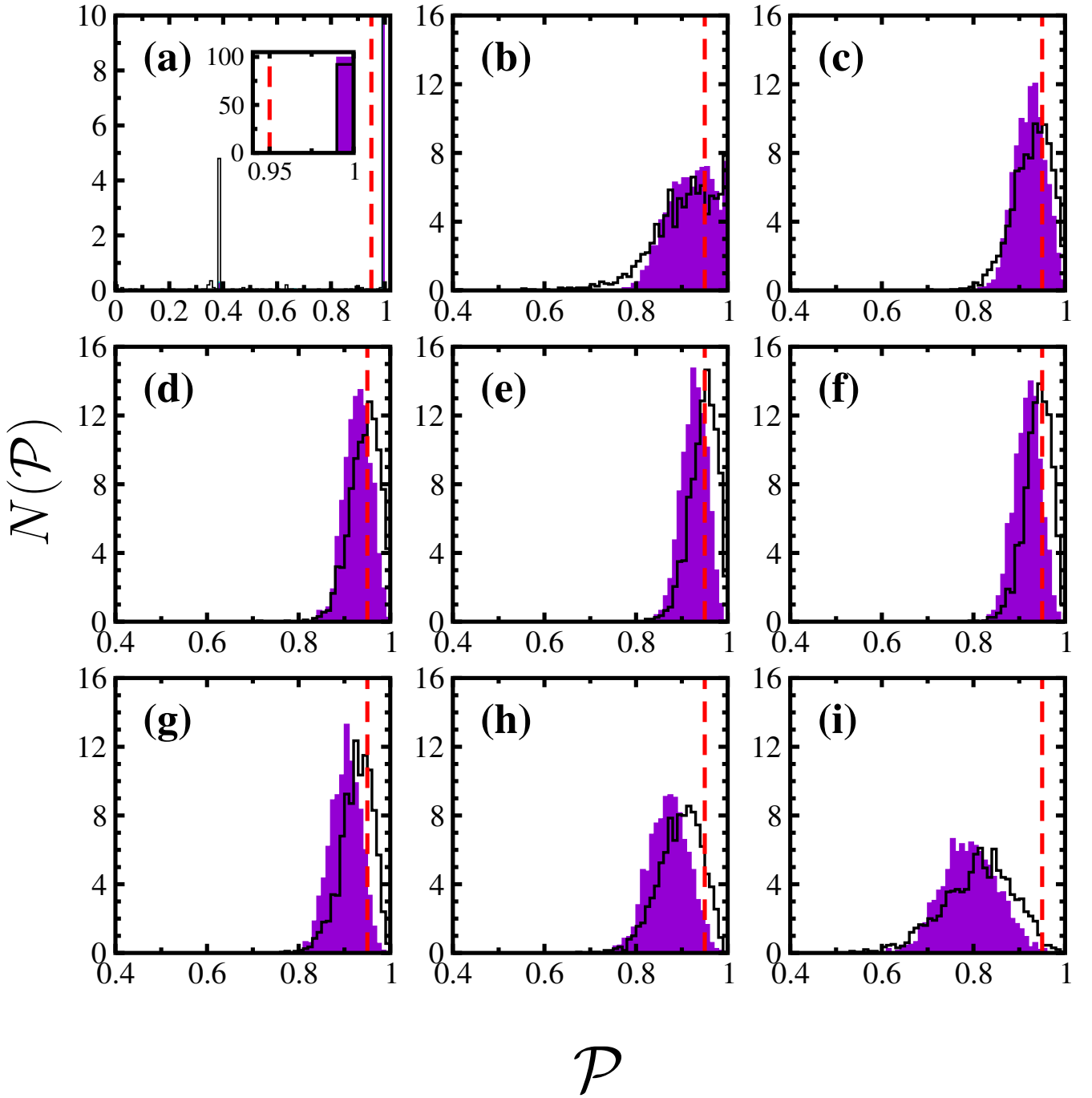


Figure 2. Same as Fig. 1 where we impose centrosymmetry or centrohermiticity for the embedded Gaussian ensembles. Note that for  $k = 1$  and  $\beta = 2$ , almost all realizations yield almost PST.

as long as the off-diagonal matrix element of the random one-body interaction matrix is different from zero. Yet, the time scale where the PST occurs may be quite long, since it is inversely proportional to the absolute value of the off-diagonal matrix element of the random one-body matrix; this explains the occurrence of other values for the efficiency, as shown in Fig. 2(a). Our results extend the validity of these statement to the broken time-reversal symmetric case and, more important, illustrate

that they are robust under certain random centrosymmetric/centrohermitian perturbations that preserve the graph structure. We emphasize the important role of centrosymmetry or centrohermiticity here: it guarantees that the diagonal matrix elements have a constant value, which results in PST among the edge states of the network.

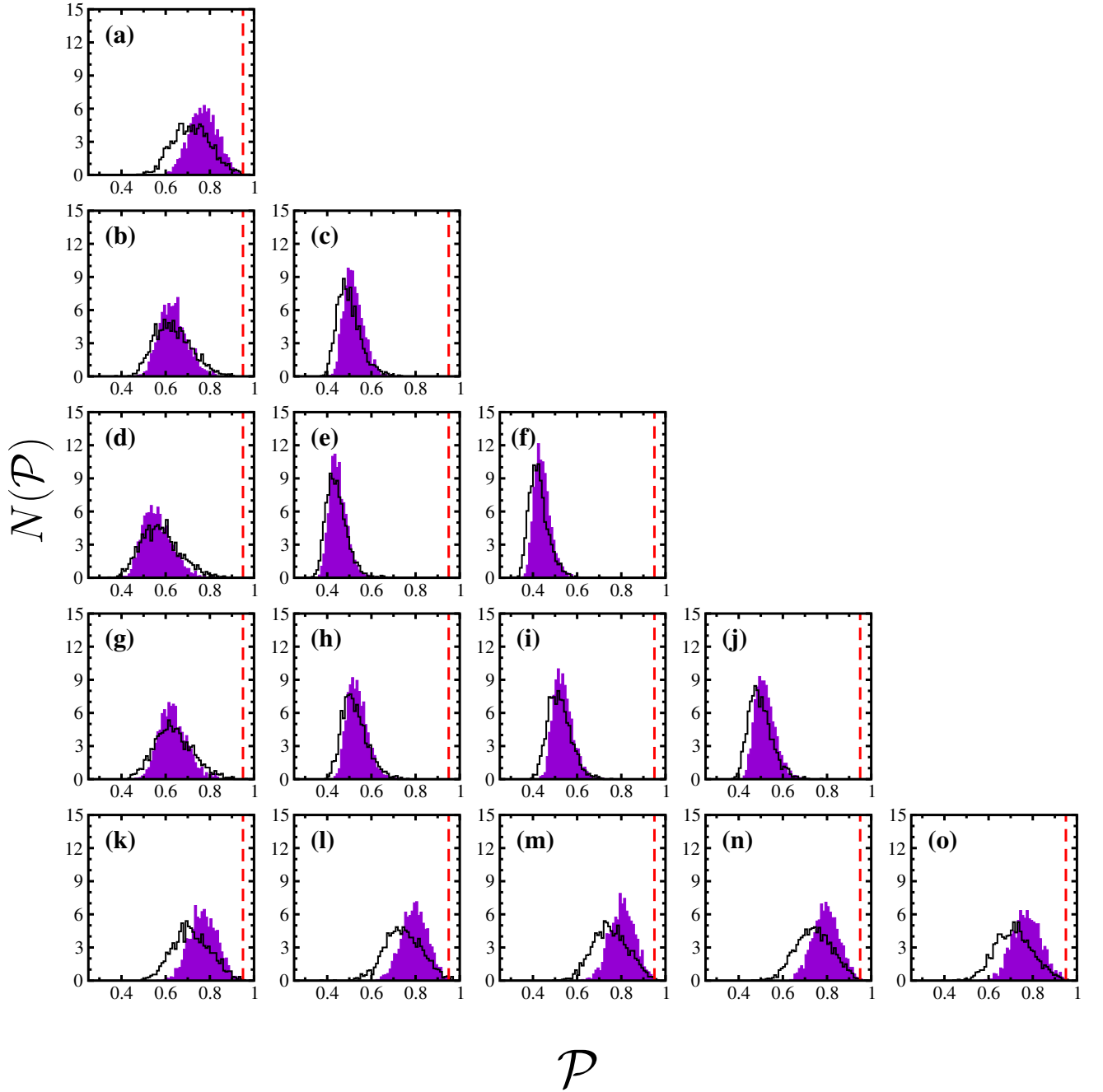


Figure 3. Normalized distributions of the best efficiencies  $\mathcal{P}$  of each realization of the the fermionic embedded Gaussian ensemble without centrosymmetry or centrohermiticity for  $l = 6$ : (a)  $k = n = 1$ , (b)  $k = 1, n = 2$ , (c)  $k = 2, n = 2$ , (d)  $k = 1, n = 3$ , (e)  $k = 2, n = 3$ , (f)  $k = n = 3$ , (g)  $k = 1, n = 4$ , (h)  $k = 2, n = 4$ , (i)  $k = 3, n = 4$ , (j)  $k = n = 4$ , (k)  $k = 1, n = 5$ , (l)  $k = 2, n = 5$ , (m)  $k = 3, n = 5$ , (n)  $k = 4, n = 5$ , (o)  $k = n = 5$ . (Rows have the same particle number  $n$ , and columns the same  $k$  value). Note that the two last rows correspond to filling factors above half-filling. Empty histograms correspond to  $\beta = 1$  and shaded (violet) to  $\beta = 2$ .

#### IV. STATISTICS OF THE TRANSPORT EFFICIENCY FOR FERMIONS

We consider now the case of the fermionic embedded Gaussian ensemble. For concreteness we analyze the case  $l = 6$ , varying the number of fermions  $n$  from 1 to 5, and

rank of interaction  $k$  also from 1 to 5. In general, the distribution of efficiencies of the ensemble is rather broad with negligible contribution to efficiencies above 95% and thus, we will focus on distribution of best efficiencies  $\mathcal{P}$  for each member of the ensemble.

We begin describing first the case where no centrosym-

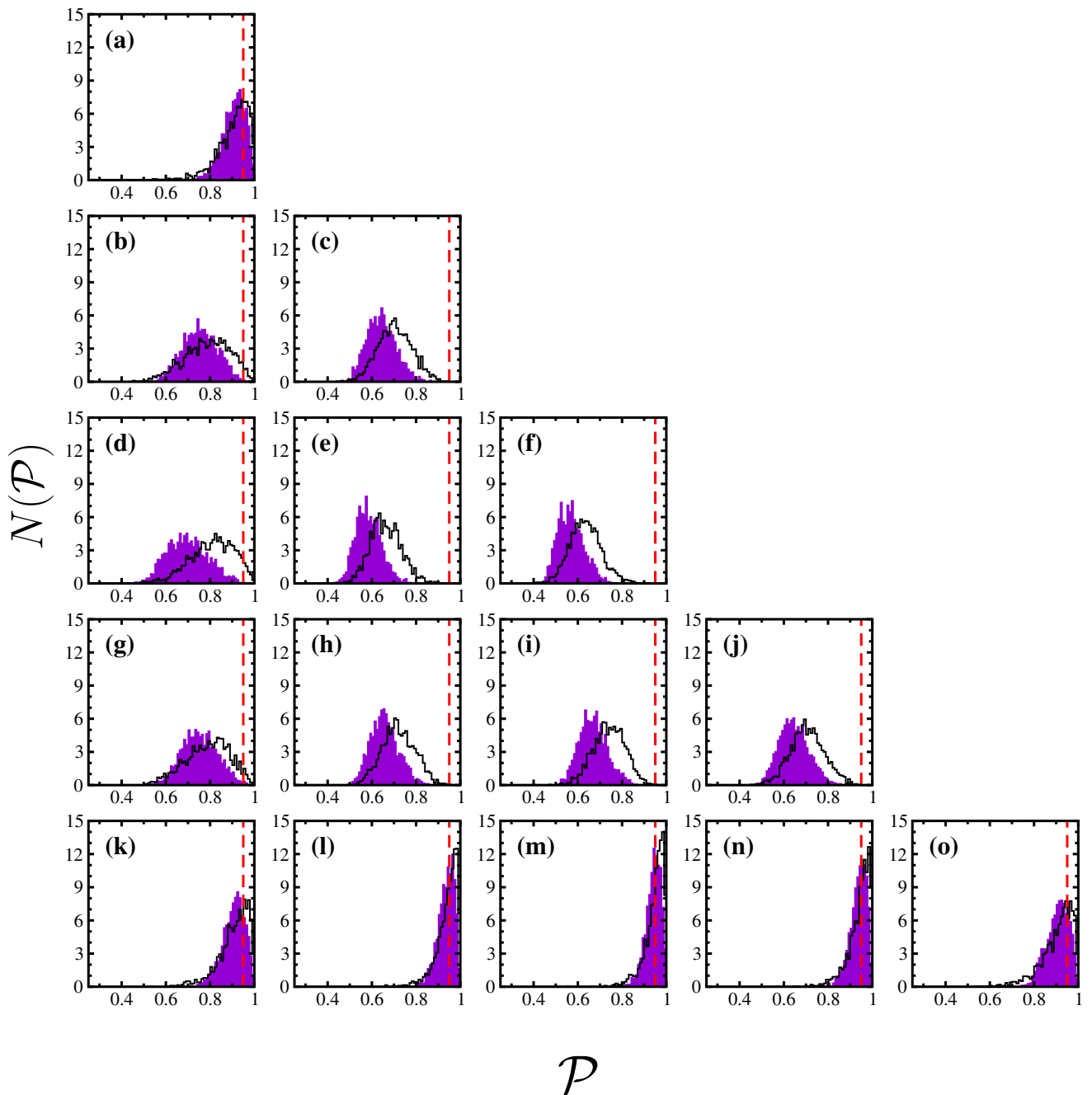


Figure 4. Same as Fig. 3 where we impose centrosymmetry or centrohermiticity in the one-particle space. Its extension to the  $k$ - and  $n$ -particle spaces is such that full centrosymmetry/centrohermiticity is achieved for odd values of  $k$  or  $n$ . Note that combining full centrosymmetry/centrohermiticity and being above half-filling, for  $k \sim l/2$  yields the best efficiencies per realization.

metry or centrohermiticity is considered. The distribution of the best efficiencies  $\mathcal{P}$  of each realization are illustrated in Fig 3. In this figure, all frames in a row have the same number of fermions  $n$ , and all frames in a column the same value of  $k$ . As before, the empty histograms correspond to  $\beta = 1$  and the filled ones to  $\beta = 2$ . In general, the lack of centrosymmetry or centrohermiticity yields poor efficiencies, with a marginal improvement

for  $\beta = 2$  compared to the  $\beta = 1$  case. Interestingly, below and at half-filling (first three rows) only the case  $k = n = 1$  marginally reaches the benchmark value, while above half-filling (last two rows of the figure) the distributions of  $\mathcal{P}$  yield better results. In particular, the case  $n = 5 = l - 1$  is the only one where some realizations achieve the benchmark value. Yet, the probability of such events is very small.

The distributions of the best efficiencies of each realization including centrosymmetry/centrohermiticity are displayed in Fig. 4. As in the bosonic case,  $\beta = 1$  seems to yield better efficiencies than the  $\beta = 2$  case. For  $k = 1$ , the distributions of  $\mathcal{P}$  for all  $n$  surpass the benchmark value, and have a non-zero probability of displaying PST, specially for odd fermion number. Yet, the  $k = 1$  case does not correspond to the overall optimal situation: the probability that  $\mathcal{P}$  is above the benchmark value is clearly larger for  $n = 5 = l - 1$  and  $2 \leq k \leq 4$ . Indeed, while we find that nearly 30% of the best efficiencies are larger than the benchmark for  $k = n = 1$  and  $\beta = 1$  (similar values are obtained for  $k = 1$  and  $k = 5$  for  $n = 5$ ), for  $k = 3$  and  $n = 5$  this percentage is close to 59%. For  $\beta = 2$  similar results are found with smaller percentages.

While centrosymmetry clearly enhances the efficiencies, its role is more involved. As we mentioned above, centrosymmetry/centrohermiticity is introduced at the one-particle level and then extended onto the  $k$ - and  $n$ -particle spaces using Eq. 4. The corresponding  $J_k$  and  $J_n$  matrices are exchange matrices or partial exchange matrices depending on the parameters  $k$  and  $n$ , and  $l$ . For the present case,  $l = 6$ , it can be shown that for odd particle-number space ( $k$  or  $n$ ) the corresponding matrix ( $J_k$  or  $J_n$ ) is a *bona fide* exchange matrix, whereas for even-values there are more than one states that are mapped onto itself, i.e., the corresponding  $J_k$  or  $J_n$  matrix is a partial exchange matrix. Therefore, independent of  $k$ , including the case where  $J_k$  is a partial exchange matrix, the  $n = 3$  and  $n = 5$  many-body Hamiltonians are fully centrosymmetric/centrohermitian. As shown in Fig. 4, full centrosymmetry or centrohermiticity, in comparison with the partial cases, yield the best efficiency distributions. When the many-body Hamiltonian is partial centrosymmetric/centrohermitian, the best distributions correspond to the  $k = 1$  case. In addition to centrosymmetry/centrohermiticity, the filling factor appears to play an important role: above half-filling the distributions of  $\mathcal{P}$  display the largest probability to yield efficiencies above the benchmark value; in terms of  $k$ , the optimal case corresponds to  $k \simeq l/2$ .

Regarding the pair of states that display the best efficiencies when centrosymmetry/centrohermiticity is imposed, as in the bosonic case, these occur among centrosymmetry related pairs,  $\psi_{n;\nu}^\dagger = J_n \psi_{n;\mu}^\dagger$ , excluding those which are mapped onto themselves by  $J_n$  whenever  $J_n$  is partial centrosymmetric. These pairs of states appear uniformly distributed, that is, there is no special pair of states that display better transport properties. This may be related to the fact that the fermionic graph is regular [15].

We have confirmed these results for  $l = 7, n = 6, k = 1, \dots, 6$  and  $l = 8, n = 7, k = 1, \dots, 7$ .

## V. CONCLUSIONS

The primary aim of the present paper is to study transport of excitations in disordered networks with random  $k$ -body interactions. This is important and certainly of interest because of possible applications in quantum communication protocols [23] and artificial solar cells [24]. Towards this end, we have studied the distribution of quantum efficiencies in disordered networks with many-body interactions, whose structure is modeled by the embedded Gaussian ensemble, considering bosons and fermions, with and without time-reversal symmetry. In particular, we studied the role played by centrosymmetry/centrohermiticity, which is defined at the one-particle space, and then extended to the  $k$ - and  $n$ -particle spaces. We have shown that (full) centrosymmetry enhances the efficiencies dramatically, being a requirement to have non-zero probability for PST; the lack of centrosymmetry/centrohermiticity yields rather poor efficiencies.

For bosons distributed in two single-particle levels, centrosymmetry is inherited in the  $k$ - and  $n$ -body spaces. In this case, PST is obtained for  $k = 1$  for almost all realizations; the fact that our computation of the efficiencies involves an upper bound for the time,  $T$ , constrains the relevant time scale for the achievement of the PST. In terms of  $k$ , the probability of having PST decays with increasing  $k$ . However, we stress that for  $k > 1$ , the best efficient scenario is when  $k \sim n/2$ ,  $n$  is the total number of bosons. With respect to the value of  $\beta$ , the results are marginally better when time-reversal symmetry is preserved. The pairs of states showing the best efficiencies are those at the edges of the network, i.e., where all bosons are in one of the two single-particle levels. Then, in this case, state transfer corresponds to the physical transport of all bosons to the other single-particle state.

For fermions, we found that full centrosymmetry/centrohermiticity of the  $n$ -particle Hamiltonian enhances considerably the best efficiencies, especially when the filling-factor is larger than  $1/2$ . We note that centrosymmetry/centrohermiticity is introduced at the one-particle level, and then extended to the  $n$ -body space. For fermions, the rank of the interaction which displays the highest probability that the best efficiency  $\mathcal{P}$  is above the benchmark value corresponds to  $k \simeq l/2$ . A clear explanation of this is left open. The pairs of states that yield the best efficiencies appear uniformly among those states linked by centrosymmetry.

Previous results have shown that random perturbations on networks with PST destroy or affect significantly this property; for details see [25, 26] and also [23] and references therein. Our results show that, despite of the random character of the  $k$ -body interactions that we have considered, certain  $n$ -body networks display good efficiencies and may attain near perfect state transfer with non-zero probability.

Our results could be exploited as new design principles of networks with good efficiency, which is preserved



under certain many-body random perturbations. For instance, considering the implementation of efficient quantum wires, it may be interesting to consider the case of filling factors that are smaller but close to one, where many-body interactions yield robustly very good efficiencies. Finally, the results in our paper open the possibility to understand the good efficiency properties experimentally observed in exciton transport in biological systems, such as the Fenna-Matthews-Olson complex [1, 8, 24].

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